34) $\frac{dy}{dx} = (1+\sqrt{2})x^{\sqrt{2}}$

36) $\frac{dy}{dx} = (1-e)x^{-e}$

p178

9, 33, 7

7) $y = xe^{2} - e^{x}$ (3x)

$y' = e^{2} - e^{x} \cdot 1$

9) $y = e^{\sqrt{x}}$

$y' = e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}$

$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

33) $y = x^{\pi}$ $\quad y = \pi^{x}$ $\quad y = a^{x}$

$y' = \pi x^{\pi-1}$ $\quad y' = \pi^{x}\ln\pi$ $\quad y' = a^{x}\ln a$

More 3.9: Derivatives of Logs

$y = \ln_e x$

$e^y = x$

$e^y \cdot y' = 1$

$y' = \frac{1}{e^y}$

$y' = \frac{1}{x}$

$$\boxed{\frac{d}{dx} \ln x = \frac{1}{x} \qquad \frac{d}{dx} \ln u = \frac{1}{u} \cdot \frac{du}{dx}}$$

ex $\frac{d}{dx} \ln\left(\sqrt{4x^2+1}\right)$ $\quad \frac{1}{2}(4x^2+1)^{-1/2}$

$= \frac{1}{\sqrt{4x^2+1}} \cdot \frac{1}{2\sqrt{4x^2+1}} \cdot 8x$

$= \frac{4x}{4x^2+1}$

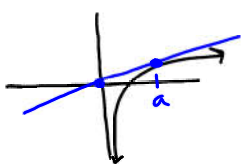
$$\frac{d}{dx}(\log_a x) = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right)$$

$$= \frac{d}{dx}\left(\frac{1}{\ln a} \cdot \ln x\right) = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$= \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\log_a u) = \frac{1}{u \ln a} \cdot \frac{du}{dx}$$

Ex) A line with slope m passes through the origin and tangent to $y = \ln x$. Find m.



Points on line: $(0,0)$ $(a, \ln a)$

$m = \frac{\ln a - 0}{a - 0} = \frac{\ln a}{a}$

$m = \frac{d}{dx}(\ln x) = \frac{1}{x}\Big|_{x=a} = \frac{1}{a}$

Set = to each other:

$\frac{\ln a}{a} = \frac{1}{a}$ so $\ln_e a = 1$

$e^1 = a$

$e = a$

Slope at e: $\frac{1}{x}\Big|_{x=e} = \boxed{\frac{1}{e}}$

Domain Issues

$f(x) = \ln(x-3)$ $\quad$ D: $x > 3$ $\quad (3, \infty)$

$f'(x) = \frac{1}{x-3}$ $\quad$ D: $x > 3$

Find $\frac{d}{dx}(x^x)$

Use Logarithmic Differentiation

$$y = x^x$$

$$\ln y = \ln x^x$$

$$\ln y = x \cdot \ln x$$

$$\frac{1}{y} \cdot y' = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$\frac{1}{y} \cdot y' = 1 + \ln x$$

$$y' = y(1 + \ln x)$$

$$y' = x^x(1 + \ln x)$$

$$y = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$$ Use log. diff.

$$\ln y = \ln \sqrt[5]{}$$

$$\ln y = \frac{1}{5}\ln\left(\frac{(x-3)^4(x^2+1)}{(2x+5)^3}\right)$$

$$\ln y = \frac{1}{5}\left[\ln(x-3)^4 + \ln(x^2+1) - \ln(2x+5)^3\right]$$

$$\ln y = \frac{1}{5}\left[4\ln(x-3) + \ln(x^2+1) - 3\ln(2x+5)\right]$$

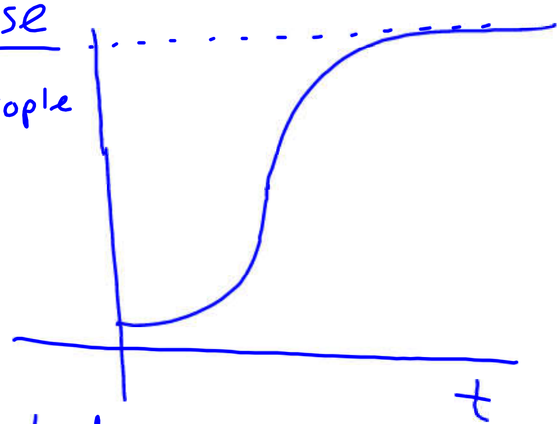
$$\ln y = \frac{4}{5}\ln(x-3) + \frac{1}{5}\ln(x^2+1) - \frac{3}{5}\ln(2x+5)$$

$$\frac{1}{y}\cdot y' = \frac{4}{5}\cdot\frac{1}{x-3} + \frac{1}{5}\cdot\frac{1}{x^2+1}\cdot 2x - \frac{3}{5}\,\frac{1}{2x+5}\cdot 2$$

$$\frac{1}{y}\cdot y' = \frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)}$$

$$y' = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)3}}\cdot\left[\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x+5)}\right]$$

Model the Spread of Disease



Flu

$$P(t) = \frac{100}{1+e^{3-t}}$$

$P(t)$ = # of patients after t days

a) initial # infected? $P(0) = \frac{100}{1+e^{3}} \approx 5$

b) How fast is it spreading after 3 days?

$$P(t) = 100\left(1+e^{3-t}\right)^{-1}$$

$$P'(t) = 100 \cdot -1\left(1+e^{3-t}\right)^{-2} \cdot e^{3-t} \cdot -1$$

$$P'(t) = \frac{100e^{3-t}}{\left(1+e^{3-t}\right)^{2}}$$

HW: p178 #6, 8, 14, 20, 30
37, 39, 43-47, 51, 64